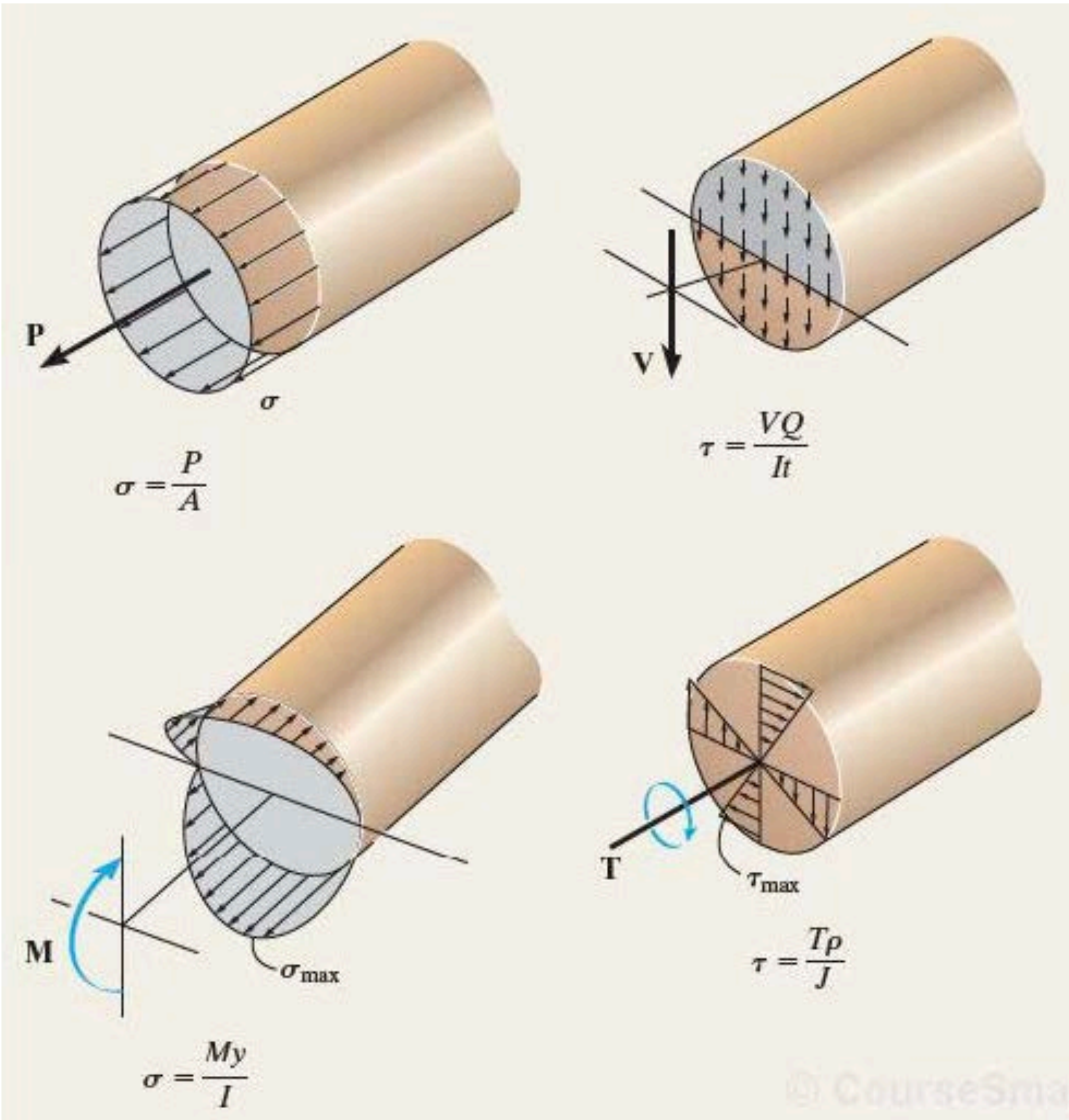
CE/ME 2420
Mechanics of Material

Reference Material

You are allowed to use this material in the Exams and Quizzes.

Α	α	Alpha	al-fah
Β	β	Beta	bay-tah
Γ	γ	Gamma	gam-ah
Δ	δ	Delta	del-tah
Ε	ε	Epsilon	ep-si-lon
Ζ	ζ	Zeta	zay-tah
Η	η	Eta	ay-tah
Θ	θ	Theta	thay-ta
Ι	ι	Iota	eye-o-tah
Κ	κ	Kappa	cap-ah
Λ	λ	Lambda	lamb-da
Μ	μ	Mu	mew
Ν	ν	Nu	new
Ξ	ξ	Xi	zee
Ο	ο	Omicron	ohm-e-cron
Π	π	Pi	pie
Ρ	ρ	Rho	roe
Σ	σ	Sigma	sig-mah
Τ	τ	Tau	taw
Υ	υ	Upsilon	oop-si-lon
Φ	φ	Phi	fie
Χ	χ	Chi	kie
Ψ	ψ	Psi	sigh
Ω	ω	Omega	o-may-gah



P
σ
$\sigma = \frac{P}{A}$
V
$\tau = \frac{VQ}{It}$
M
σ_{max}
$\sigma = \frac{My}{I}$
T
τ_{max}
$\tau = \frac{T\rho}{J}$

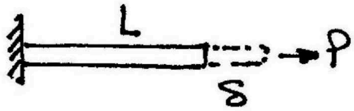
Axial or Normal

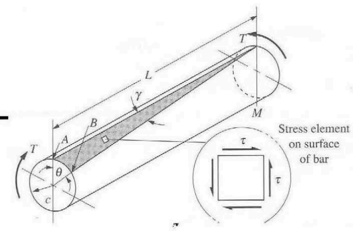
$$\sigma = \frac{P}{A}$$

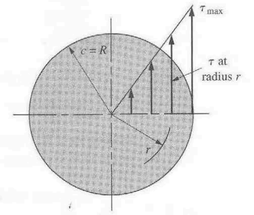
$$\epsilon = \frac{\delta}{L}$$

$$E = \frac{\sigma}{\epsilon}$$

$$\delta = \frac{PL}{AE}$$

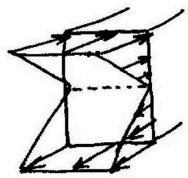


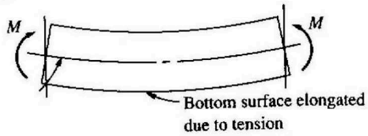




Torsion $\tau = \frac{T r}{J}$

$$\theta = \frac{TL}{JG} \qquad J = \frac{\pi}{32} D^4$$

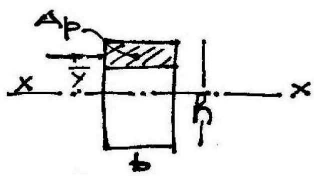


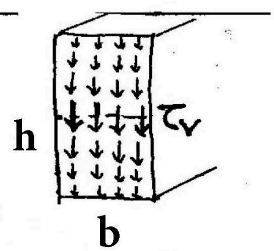


Bending $\sigma = \frac{Mc}{I}$

$$\frac{I}{c} = S = \text{section modulus}$$

Shear $\tau = \frac{VQ}{It}$; $Q = A_p \bar{y}$





$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$G = \frac{E}{2(1+\nu)}$$

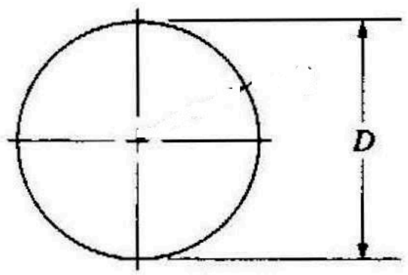
$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

A–1 Properties of areas.*

Circle

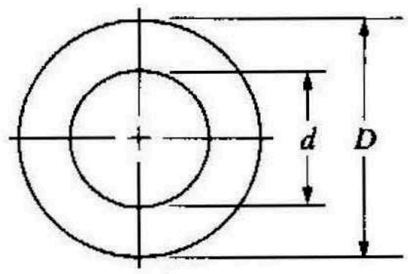


$$A = \frac{\pi D^2}{4}$$

$$I = \frac{\pi D^4}{64} \qquad J = \frac{\pi D^4}{32}$$

Circumference $= \pi D$

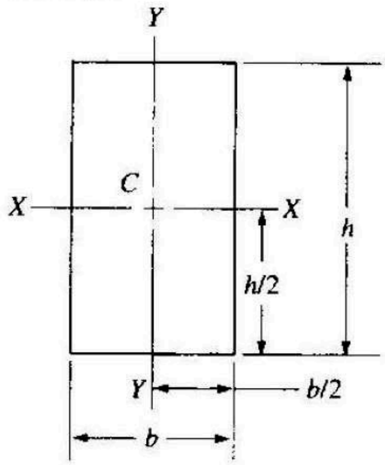
Hollow circle (tube)



$$A = \frac{\pi(D^2 - d^2)}{4}$$

$$I = \frac{\pi(D^4 - d^4)}{64} \qquad J = \frac{\pi(D^4 - d^4)}{32}$$

Rectangle

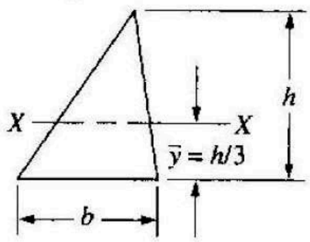


$$A = bh$$

$$I_x = \frac{bh^3}{12}$$

$$I_y = \frac{hb^3}{12}$$

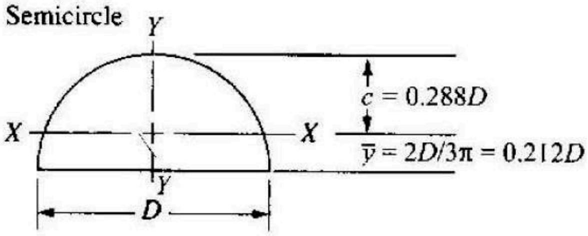
Triangle



$$A = \frac{bh}{2}$$

$$I_x = \frac{bh^3}{36}$$

Semicircle

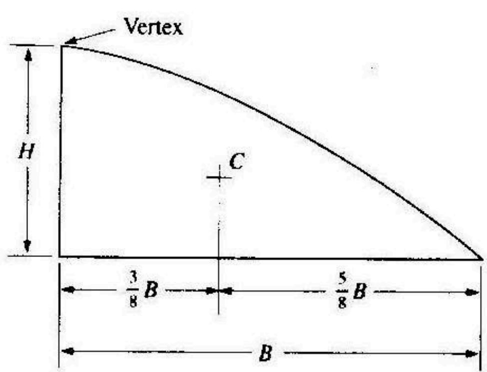


$$A = \frac{\pi D^2}{8}$$

$$I_x = 0.00686D^4$$

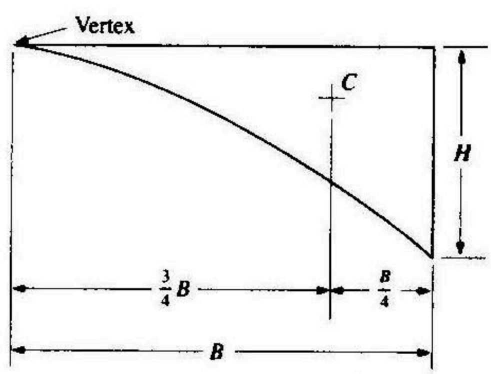
$$I_y = 0.0245D^4$$

Area under a second-degree curve



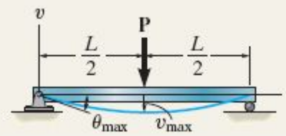
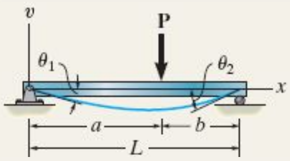
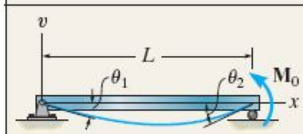
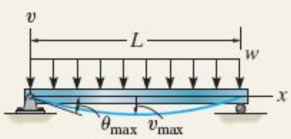
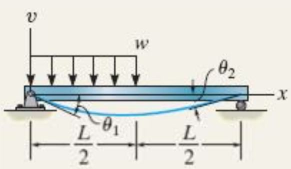
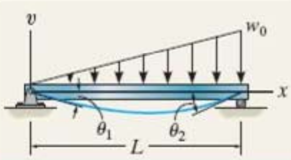
$$A = \frac{2}{3}BH$$

Area over a second-degree curve



$$A = \frac{1}{3}BH$$

Simply Supported Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \dfrac{-PL^2}{16EI}$	$v_{max} = \dfrac{-PL^3}{48EI}$	$v = \dfrac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \dfrac{-Pab(L+b)}{6EIL}$ $\theta_2 = \dfrac{Pab(L+a)}{6EIL}$	$v\Big\|_{x=a} = \dfrac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \dfrac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$\theta_1 = \dfrac{-M_0 L}{6EI}$ $\theta_2 = \dfrac{M_0 L}{3EI}$	$v_{max} = \dfrac{-M_0 L^2}{9\sqrt{3}EI}$ at $x = 0.5774L$	$v = \dfrac{-M_0 x}{6EIL}(L^2 - x^2)$
	$\theta_{max} = \dfrac{-wL^3}{24EI}$	$v_{max} = \dfrac{-5wL^4}{384EI}$	$v = \dfrac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$
	$\theta_1 = \dfrac{-3wL^3}{128EI}$ $\theta_2 = \dfrac{7wL^3}{384EI}$	$v\Big\|_{x=L/2} = \dfrac{-5wL^4}{768EI}$ $v_{max} = -0.006563\dfrac{wL^4}{EI}$ at $x = 0.4598L$	$v = \dfrac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \dfrac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2 x - L^3)$ $L/2 \le x < L$
	$\theta_1 = \dfrac{-7w_0 L^3}{360EI}$ $\theta_2 = \dfrac{w_0 L^3}{45EI}$	$v_{max} = -0.00652\dfrac{w_0 L^4}{EI}$ at $x = 0.5193L$	$v = \dfrac{-w_0 x}{360EIL}(3x^4 - 10L^2x^2 + 7L^4)$

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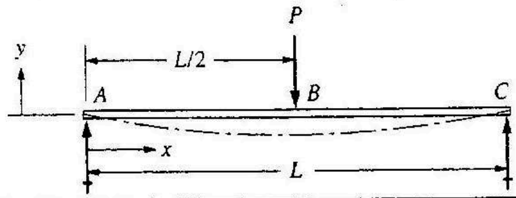
A–22 Beam deflection formulas—simply supported beams.

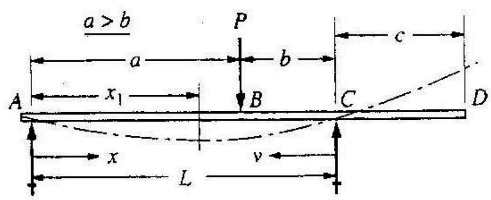


(a)

$$y_B = y_{max} = \frac{-PL^3}{48EI} \quad \text{at center}$$

Between A and B:

$$y = \frac{-Px}{48EI}(3L^2 - 4x^2)$$



(b)

$$y_{max} = \frac{-Pab(L + b)\sqrt{3a(L + b)}}{27EIL}$$

at $x_1 = \sqrt{a(L + b)/3}$

$$y_B = \frac{-Pa^2b^2}{3EIL} \quad \text{at load}$$

Between A and B (the longer segment):

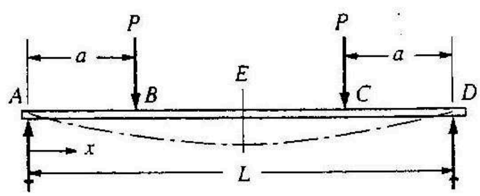
$$y = \frac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$$

Between B and C (the shorter segment):

$$y = \frac{-Pav}{6EIL}(L^2 - v^2 - a^2)$$

At end of overhang at D:

$$y_D = \frac{Pabc}{6EIL}(L + a)$$



(c)

$$y_E = y_{max} = \frac{-Pa}{24EI}(3L^2 - 4a^2) \quad \text{at center}$$

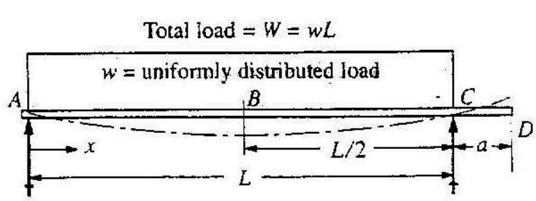
$$y_B = y_C = \frac{-Pa^2}{6EI}(3L - 4a) \quad \text{at loads}$$

Between A and B:

$$y = \frac{-Px}{6EI}(3aL - 3a^2 - x^2)$$

Between B and C:

$$y = \frac{-Pa}{6EI}(3Lx - 3x^2 - a^2)$$



(d)

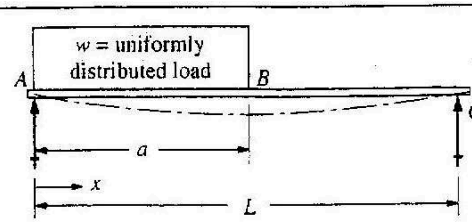
$$y_B = y_{max} = \frac{-5wL^4}{384EI} = \frac{-5WL^3}{384EI} \quad \text{at center}$$

Between A and B:

$$y = \frac{-wx}{24EI}(L^3 - 2Lx^2 + x^3)$$

At D at end:

$$y_D = \frac{wL^3a}{24EI}$$



(e)

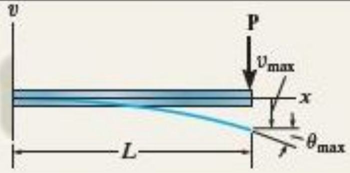
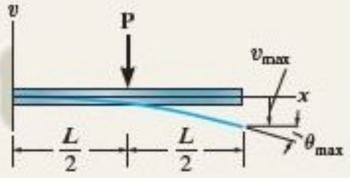
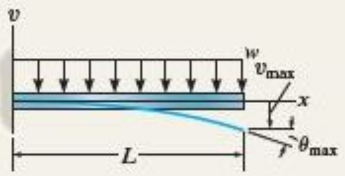
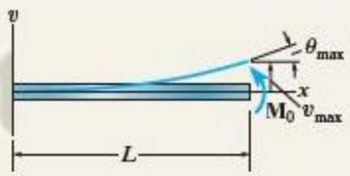
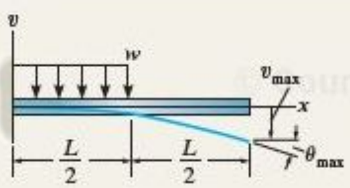
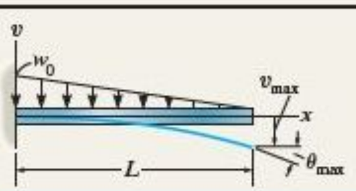
Between A and B:

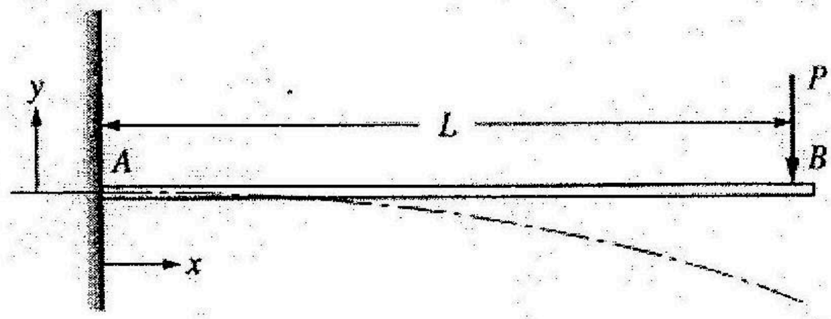
$$y = \frac{-wx}{24EIL}[a^2(2L - a)^2 - 2ax^2(2L - a) + Lx^3]$$

Between B and C:

$$y = \frac{-wa^2(L - x)}{24EIL}(4Lx - 2x^2 - a^2)$$

Cantilevered Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \dfrac{-PL^2}{2EI}$	$v_{max} = \dfrac{-PL^3}{3EI}$	$v = \dfrac{-Px^2}{6EI}(3L - x)$
	$\theta_{max} = \dfrac{-PL^2}{8EI}$	$v_{max} = \dfrac{-5PL^3}{48EI}$	$v = \dfrac{-Px^2}{6EI}\left(\tfrac{3}{2}L - x\right) \quad 0 \le x \le L/2$ $v = \dfrac{-PL^2}{24EI}\left(3x - \tfrac{1}{2}L\right) \quad L/2 \le x \le L$
	$\theta_{max} = \dfrac{-wL^3}{6EI}$	$v_{max} = \dfrac{-wL^4}{8EI}$	$v = \dfrac{-wx^2}{24EI}(x^2 - 4Lx + 6L^2)$
	$\theta_{max} = \dfrac{M_0L}{EI}$	$v_{max} = \dfrac{M_0L^2}{2EI}$	$v = \dfrac{M_0x^2}{2EI}$
	$\theta_{max} = \dfrac{-wL^3}{48EI}$	$v_{max} = \dfrac{-7wL^4}{384EI}$	$v = \dfrac{-wx^2}{24EI}\left(x^2 - 2Lx + \tfrac{3}{2}L^2\right)$ $0 \le x \le L/2$ $v = \dfrac{-wL^3}{192EI}(4x - L/2)$ $L/2 \le x \le L$
	$\theta_{max} = \dfrac{-w_0L^3}{24EI}$	$v_{max} = \dfrac{-w_0L^4}{30EI}$	$v = \dfrac{-w_0x^2}{120EIL}(10L^3 - 10L^2x + 5Lx^2 - x^3)$

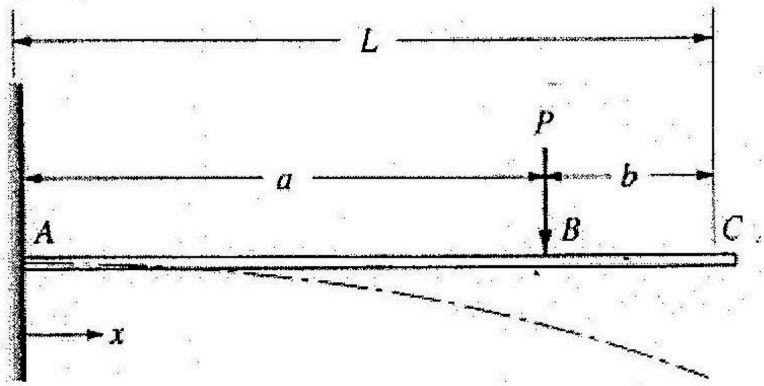


At B at end:

$$y_B = y_{max} = \frac{-PL^3}{3EI}$$

Between A and B:

$$y = \frac{-Px^2}{6EI}(3L - x)$$



At B at load:

$$y_B = \frac{-Pa^3}{3EI}$$

At C at end:

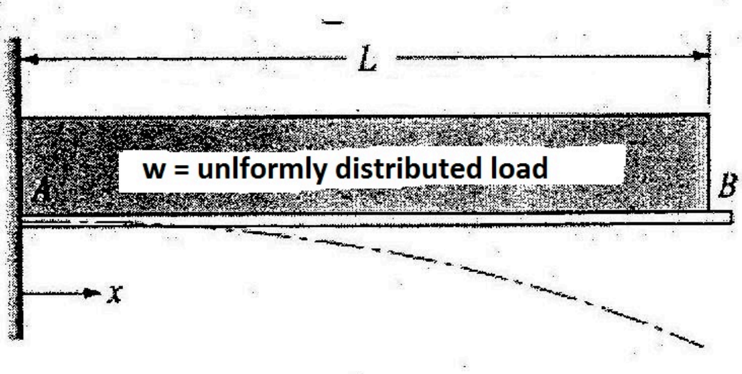
$$y_C = y_{max} = \frac{-Pa^2}{6EI}(3L - a)$$

Between A and B:

$$y = \frac{-Px^2}{6EI}(3a - x)$$

Between B and C:

$$y = \frac{-Pa^2}{6EI}(3x - a)$$



At B at end:

$$y_B = y_{max} = \frac{-wL^4}{8EI}$$

Between A and B:

$$y = \frac{-wx^2}{24EI}(x^2 - 4Lx + 6L^2)$$

Page 443 – 447 (Hibbeler)

Any Plane

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal Plane No Shear

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

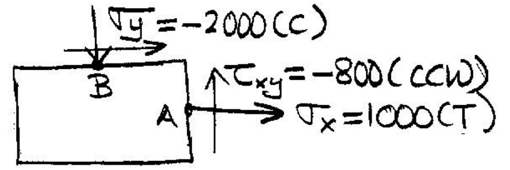
$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x-\sigma_y)/2}$$

Max Shear Plane + σ_{avg}

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

at $$\tan 2\theta_s = \frac{-(\sigma_x-\sigma_y)/2}{\tau_{xy}}$$

$$\sigma_{avg} = \frac{\sigma_x+\sigma_y}{2}$$



$$O = \frac{1}{2}(\sigma_x + \sigma_y) = -500$$

$$a = \frac{1}{2}(\sigma_x - \sigma_y) = 1500$$

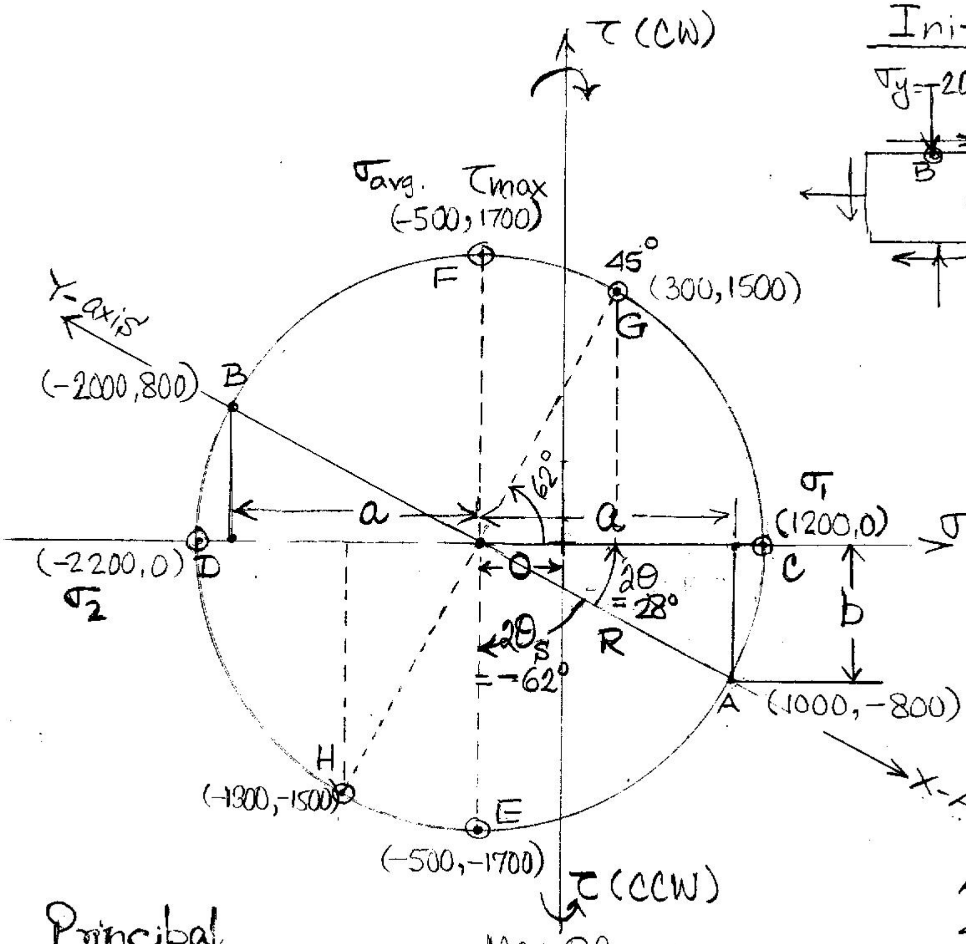
$$b = \tau_{xy} = -800$$

$$2\theta = \tan^{-1}\left(\frac{b}{a}\right) = -28^\circ$$

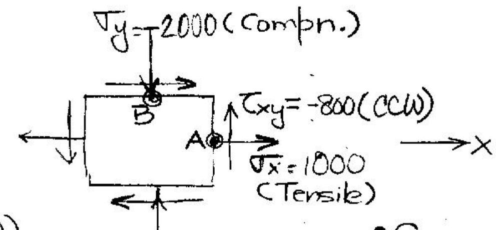
$$R = \sqrt{a^2 + b^2} = \sqrt{1500^2 + 800^2} = 1700$$

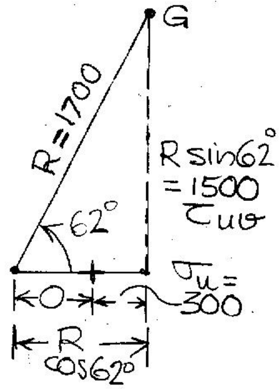
$$\sigma_1 = O + R = 1200$$

$$\sigma_2 = O - R = -2200$$



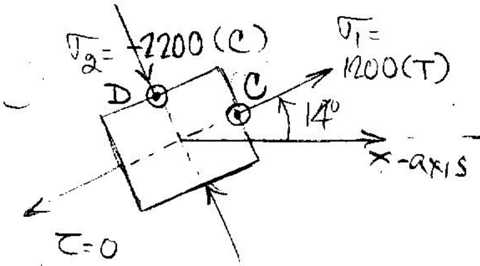
Initial



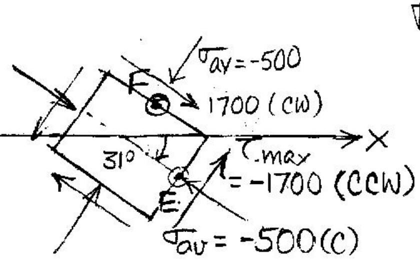


45° Plane

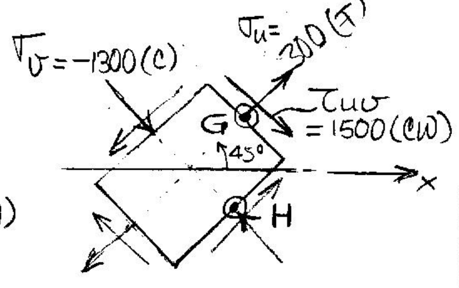
Principal



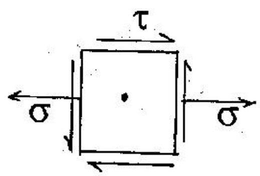
Max. Shear



Any given angle
45° CCW



$$\varepsilon_n = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos(2\phi) + \frac{\gamma_{xy}}{2}\sin(2\phi)$$



Maximum Shear Stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

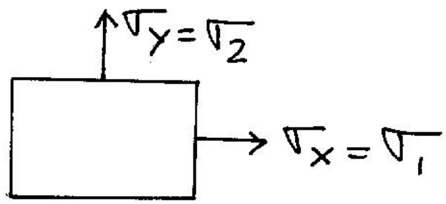
von Mises Stress

$$\sigma_m = \sqrt{\sigma^2 + 3\tau^2}$$

$\sigma_m = \sigma_{\text{Von Mises}}$

$$\sigma_m = \left(\frac{1}{\sqrt{2}}\right)\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$$

Plane Stress: $\sigma_m = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$



Cyl: $\sigma_H = \frac{pD_m}{2t}$, $\sigma_L = \frac{pD_m}{4t}$, $\sigma_m = \sigma_H\sqrt{\frac{3}{4}}$

$\tau_{max} = \sigma_H/2$

Sph: $\sigma_H = \frac{pD_m}{4t}$, $\sigma_m = \sigma_H$, $\tau_{max} = \sigma_H/2$

$\frac{D_m}{t} \geq 20$ Thin Shell

Truss Deflections:

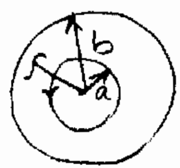
$$\frac{1}{2}P\Delta = \sum \frac{N^2L}{2AE} \quad \text{(Energy)}$$

$$\Delta = \frac{1}{AE}\sum N\left(\frac{\partial N}{\partial P}\right)L \quad \text{(Castigliano)}$$

Thick Walled Cylinder

Thick-walled cylinder	Stress at point r	Maximum Stress
Longitudinal	$\sigma_1 = \dfrac{pa^2}{b^2 - a^2}$	$\sigma_1 = \dfrac{pa^2}{b^2 - a^2}$ (uniform throughout wall)
Hoop (tangential)	$\sigma_2 = \dfrac{pa^2(b^2 + r^2)}{r^2(b^2 - a^2)}$	$\sigma_2 = \dfrac{p(b^2 + a^2)}{b^2 - a^2}$ (at inner surface)
Radial	$\sigma_3 = \dfrac{-pa^2(b^2 - r^2)}{r^2(b^2 - a^2)}$	$\sigma_3 = -p$ (at inner surface)

$$\frac{D_m}{t} < 20 \text{ (Thick Wall)}$$



Column Buckling Formulae

Euler's

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$\frac{P_{cr}}{A} = \frac{\pi^2 E}{(L_e / r)^2}$$

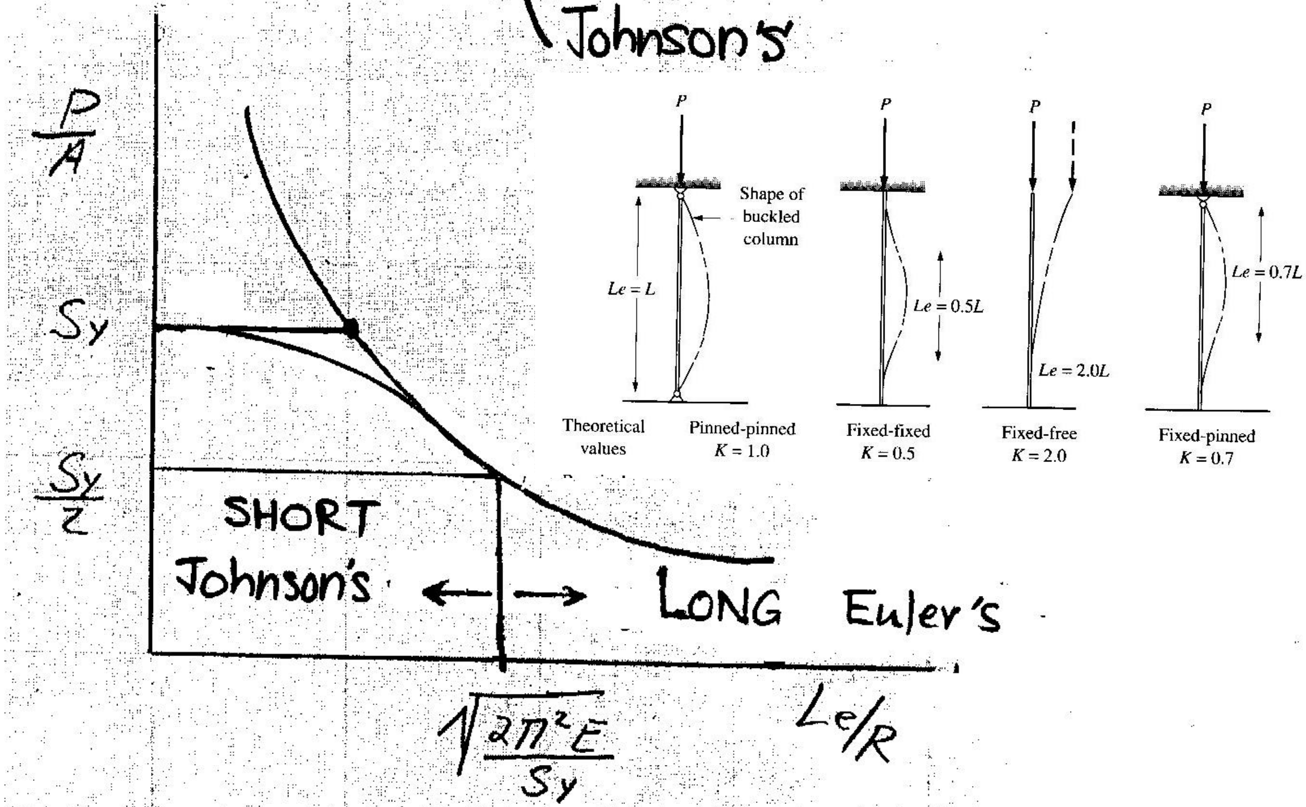
$$P_{cr} = AS_y\left[1 - \frac{S_y (L_e / r)^2}{4\pi^2 E}\right]$$

Johnson's

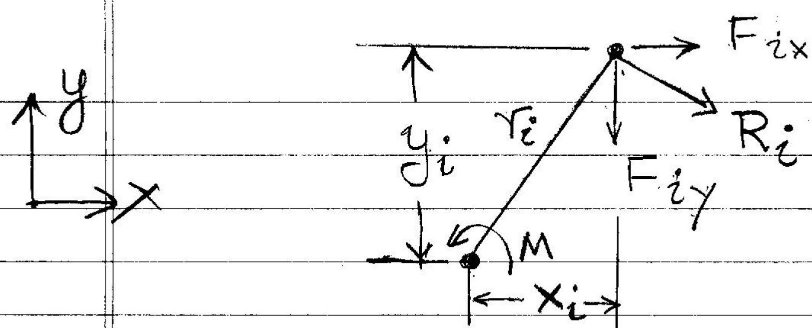
$$r = \sqrt{\frac{I}{A}}$$

$$SR = \frac{L_e}{r}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_y}}$$



Eccentrically Loaded Bolted Joints



$$F_{ix} = \frac{M\,y_i}{\sum(x_i^2 + y_i^2)} = K y_i$$

$$F_{iy} = \frac{M\,x_i}{\sum(x_i^2 + y_i^2)} = K x_i$$

$$K = \frac{M}{\sum(x_i^2 + y_i^2)}$$